Air and Space this Week

Item of the Week

Surveying With the Stars

Originally appeared January 4, 2021

KEY WORDS: Parallax   Trigonometry   Henderson   Bessel   Struve   parsec   Aristarchus Eratosthenes

No, it's not a boring new reality TV show for mathematically-inclined little-known geographers.

Humans have always wondered about the stars: what they are, how they shine, and how far they are away. The answer to the "how far away" question, at least for some stars, has known for two hundred years, using a technique conceptually for a millennium, and practiced routinely by land surveyors for centuries.

A them that shows up in A+StW Items from time-to-time is the important synergy between scientific observation and the technology that enables it. This is another of those times.

Hold out your arm in front of you, with your thumb pointing up. Close one eye, and move your thumb to cover a spot on the wall away from you. Without moving your arm/thumb, open the closed eye and close the open one. Your thumb will appear to jump laterally a bit relative to the background. It's not an optical illusion, the jump is due to the fact your eyes are looking at your thumb from exactly the same location. Your eye-brain combination is very good at converting the slightly offset images from your eyes into one view, with the degree offset being proportional to the eye-thumb distance. This is how we have depth perception.

Some of the early Greek astronomers, notably Aristarchus of Samos, figured out that the Earth was not the Center of the Universe, but rather it circled the Sun, and that the Earth was a rotating sphere, causing the day/night cycle. His insight was amazing, he had the basics absolutely correct, 1500 years before Copernicus. Much of his work has been lost to history, but just imagine what it might have been like if his views had prevailed…. We'd have been on the Moon before 969 CE, not 1969!

[Aside: Aristarchus was not the only excellent scientist/mathematician that hailed from Samos. A guy named Pythagoras was born there, too. There must have been something in the water there that made one think of triangles or something. Pythagoras lived from ~570-500 BCE; Aristarchus from 310-230 BCE.]

One of the best arguments against Aristarchus' view was an indirect one – use the idea to be tested to make a logical prediction, then observe to see if that prediction is validated by truth. If not, then the idea is probably incorrect. If the Earth were, in fact, circling the Sun, then one would expect nearer stars would, over the course of a year, show the same lateral jump relative to more distant stars, that your thumb did in our little test above. Aristarchus and others knew of this effect, and looked for it, but the stars seemed fixed in place. Therefore, Aristarchus could not be correct, since his model would predict such a shift. Unless…. The only way Aristarchus could be right would be if the stars were really, really far away.

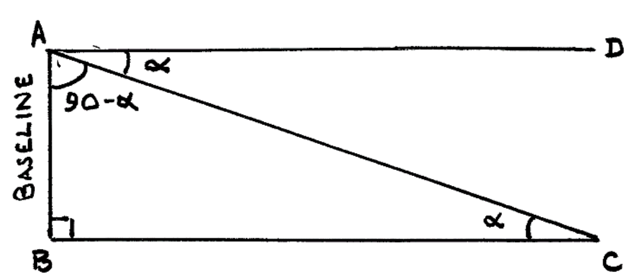
Let's try that thumb thing again. Observe how big the jump is when your thumb is held at arm's length, then move your thumb closer to your eye and check the jump again. The closer your thumb is to your eye, the larger the jump will appear to be. If you could put your thumb twice the length of your arm/eye distance, the jump would be even smaller. And if you couldn't see a jump at all, your thumb would have to be much farther away from your eye than the mere length of your arm.

The state of observational technology in Aristarchus' time did not allow for a proper test of his hypothesis of enormous stellar distances. The telescope would not be invented for almost 2000 years after Aristarchus' death. Early telescopes and primitive mountings were not up to the task, either (although Galileo made some very profound astronomical observations in the first two years any sort of telescope was available: the discovery of spots on the Sun (therefore, the Sun was not itself, divine), the discovery of the major moons of Jupiter (proving that the Earth was not the Center of All Motion in the Universe), and the full range of lunar-like phases of Venus (known by Copernicus to be the key observational test of geocentricism versus heliocentricism)).

However, by the 1830s, telescope technology had advanced greatly, and stellar positions could be determined with a high degree of precision. The basic idea of Aristarchus could finally be put to a proper astronomical test. The race was on; who would be the first to measure the first distance to a star?

Trigonometric Interlude

Surveyors have long known how to measure the distance to a distant object without having to unspool a very long and inconvenient tape measure. They use the mathematical branch of trigonometry and an interesting and useful property of right triangles, illustrated below.



Survey set-up to determine the width of a river without crossing it.

Surveyors determine where the distance across the river should be located, line segment BC. They set up a baseline, perpendicular to line segment BC, on their side of the river, line segment AB. Angle ABC was set up as a right angle. A simple angle-measuring device allows the surveyors to measure CAB = 90° - angle α. Note that because line segments AD and BC are parallel, and because line segment AC cuts both, and that "alternate interior angles are congruent," angle α, called the *parallax angle*, appears twice in the diagram.

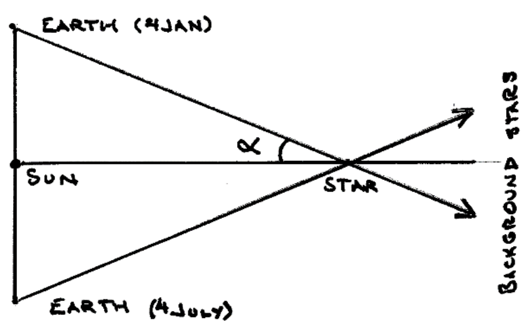
The length of the baseline AB can be measured by tape. Recall that the *tangent* of an angle is the ratio of the opposite side, in this case AB, to the adjacent side, BC, which is what we want to measure.

$$\tan \alpha = AB/BC$$

Multiply both top and bottom on the right side by BC. Then divide both sides by tan a

$$BC = AB / \tan \alpha$$

Now, instead of surveyors and a river to be measured, let's change them to astronomers seeking the distance to the stars. The baseline is the radius of the Earth's orbit. The parallax angles are tiny, but by 1830 telescopes were advance enough that the parallax of the nearest stars might be able to be determined….



Note that the parallax angle is greatly exaggerated iagram, for clarity. The largest stellar parallax found to date is less than one second of arc!

A number of astronomers were interested in stellar distances, but three were in a position to make a serious attempt:

- Friedrich Wilhelm Bessel, an outstanding mathematician, who focused his attention on a faint star, 61 Cygni, because it was found to have a large *proper motion*, movement with respect to more distant starts behind it, unrelated to parallax. Such rapid movement implies the star is close.
- Friedrich Georg Wilhelm von Struve, an expert on double stars. Struve strove (sorry, I couldn't resist) to determine the parallax of the bright star Vega, the *lucida* of the constellation Lyra.
- Thomas Henderson, the first Astronomer Royal of Scotland, who had spent two years observing the southern skies from Cape Town in Africa (1831-1833). He focused on the bright star Rigel Kentaurus (aka Alpha Centauri), which was not visible from northern latitudes.

The Royal Astronomical Society (RAS) offered a Gold Medal each year for the greatest contribution to astronomy, a very prestigious award. The first accurate stellar distance determined from its parallax would surely win the Medal. The race was on! Somewhat slowly.

Henderson actually made the first successful parallax measurement. He was an excellent, meticulous observer, and he was measuring the closest star of the three seekers. It was a difficult measurement; the parallax shift was only about one second of arc! As a contrast, the minimum separation between Jupiter and Saturn in last week's superb conjunction was six times that! But, for a variety of reasons, he was slow to process his data and publish his results, which did not reach the RAS until **January 9**, 1839. His bad.

NOTE: Astronomers routinely use the light year as a standard astronomical distance unit; the distance light, traveling at 186,282 or so miles per second, will go in one Earth year. Another standard astronomical distance is the parsec, short for "parallax second," the distance from Earth an object would be if its parallax were one second of arc. A parsec is about 3.26 light-years. Henderson was a little off; Rigel Kentaurus is just over 4 light-years away.

Bessel beat Henderson to press by two months, and got the credit, and the Gold Medal (1841). He'd been right in choosing 61 Cygni – its high proper motion was due to it being very "close" to Earth. The parallax angle he measured was about a third that of Rigel Kentaurus, giving a distance of ~3.5 parsecs or ~11 light-years.

Struve had a tougher row to hoe. The star he picked, Vega, is about 25 light-years away, with a correspondently-small (and more difficult to measure) parallax. Further, he did publish his results, but in Latin in an obscure journal.

The parallax method of determining astronomical distances is very reliable, at least for stars within ~100 light-years of Earth or so. The more distant the star, the smaller its parallax, and the more significant observational errors in measurements become.

TODAY

Advances in observing technology, spurred in part by the desire to measure stellar distances more precisely, are helping us use the parallax method more broadly than ever before. One example involves the Very Long Baseline Array, a set of 10 radio telescopes around the Earth, linked together electronically. The baseline in this case, essentially the diameter of the Earth, is much smaller than Earth's orbit, but the radio telescopes use interferometry to make extremely precise measurements.

Allow me to reprise the following Astronomy news item from a few weeks ago:

"The BeSSeL Project: No, I'm not having a problem with my auto-correct, I'm talking about the **B**ar and **S**piral **S**tructure **L**egacy Survey, an "e"-challenged observing program to use the Very Long Baseline Array to determine the distance to strong radio sources in the Milky Way arms that are invisible to the Gaia space mission because of obscuration by interstellar dust. As part of the research, BeSSeL astronomers revisited the earliest observations, data, and parallax determinations by Friedrich Wilhelm Bessel (61 Cygni), Friedrich Wilhelm von Struve (Vega), and Thomas Henderson (Rigel Kentaurus)."

Bessel won more than the RAS Gold Medal for publishing the first stellar distance. The Project was tortuously-named in his honor for being first, too.

Another Nod to the Ancients

This falls under the heading of "Surveying with the Stars," too, because the Sun is a star!

Aristarchus was a visionary when it came to understanding the Solar System, and he used basic trigonometry in the process. But he was not the only scientist "back in the day" who used basic trigonometry to make fundamental discoveries.

My favorite example is Eratosthenes, one of the string of Librarians of the Great Library at Alexandria, with many interests in the natural sciences. The Greeks believed that the Earth was a sphere, not flat, based on their observations of things like the curvature of the Earth's shadow on the Moon during a lunar eclipse, and the manner in which a ship appears to sail "over the horizon," with its hull disappearing before the sails.

Eratosthenes took his analysis to the next level. He reasoned that, if the Earth were a sphere, and the Sun so far away that its rays came to Earth parallel to one another, then it should be a simple matter to determine the size of the Earth, at least in principle if not in practice.

He had grown up on what is now the coast of Libya. It was a well-known observation there that on the longest day of the year, the Sun would pass directly overhead at local noon, because the deep town well, dug as a vertical cylinder, was fully-illuminated at that particular moment and no other. Later, Eratosthenes noticed that in Alexandria at noon on the longest day, a vertical obelisk still cast a shadow, indicating that the Sun was NOT directly overhead. He measured the angle between the local zenith at Alexandria and the Sun's position that day using trigonometry, then used the axiom (recall your high school geometry), when parallel lines are cut by a diagonal, "alternate interior angles are congruent." The diagram here will help. This gave him the angle between the vector going from Earth's center to the Libyan coast, and the vector going from Earth's center and Alexandria. All he needed to calculate the Earth's circumference was the distance between his home town and Alexandria, no mean fe**e**t (bad pun alert). The two places are quite a distance apart, but surveying was well-developed at that point in time, using simple angle measuring devices and bematists, people specially trained to have a precise and consistent stride length. He had a team walk out the distance, finished the calculation, and came up with a value within 10% of the circumference we know today with much better instrumentations. For more on the story of Eratosthenes and the circumference of the Earth, see:

- https://en.wikipedia.org/wiki/Earth%27s_circumference
- https://javalab.org/en/measuring_the_earth_en
- http://scihi.org/eratosthenes

- http://www.astronomy.ohio-state.edu/~thompson/1101/lecture_aristarchus.html
- Nicastro, Nicholas, 2008, *Circumference: Eratosthenes and the Ancient Quest to Measure the Globe*, New York: St. Martin's Press; ISBN-10: 0-312-37247-7.

Last Edited on 03 January 2021